

Instructions:

Show your work and clearly mark

your answers.

Problem 1. Determine whether each equation defines y as a function of x . If the equation is not a function, then **prove it** by picking a value of x and finding at least two corresponding values of y .

(1) $y = |x^7 - 1|$

(2) $x = |y - 13|$

(3) $x^3 + 5y^5 = 13$

(4) $x = 20 - y^2$

Problem 2. For each of the following functions, compute a) $f(3)$, b) $f(x + 2)$, and c) $\frac{f(x+h)-f(x)}{h}$:

(1) $f(x) = 5x - 3$

(2) $f(x) = x^2 - 4x + 1$

Problem 3. Determine whether the following functions are even, odd, or neither.

(1) $f(x) = 2x^3 + 7x$

(2) $f(x) = 6x^6 - 3x^4 + x^2 - 2$

(3) $f(x) = \frac{6x^5}{3x^3+7x}$

(4) $f(x) = \frac{4x^3}{x^2+1}$

Problem 4. Consider the following piecewise function:

$$f(x) = \begin{cases} 5x - 2 & \text{if } x \leq -2 \\ x^2 & \text{if } -2 < x \leq 3 \\ |x - 1| & \text{if } x > 3 \end{cases}$$

Evaluate the following:

(1) $f(-2)$

(2) $f(3)$

(3) $f(4)$

Problem 5. Let $f(x)$ be the graph below:

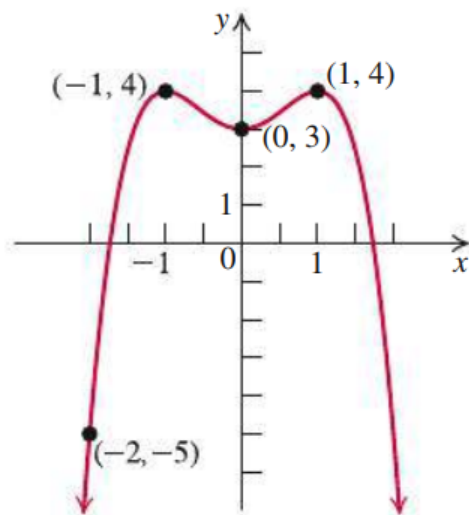


FIGURE 0.1

- (1) Determine the intervals where the function is increasing and decreasing.
- (2) Draw the graph of $f(x + 1) - 2$. Make sure to label the key points.

Problem 6. Find the domain of the following functions:

$$(1) f(x) = \sqrt{2x - 18}$$

$$(2) f(x) = 7x^4 - 2x^3 - 4x^2 + x - 10$$

$$(3) f(x) = \frac{x-2}{x^2-4x-21}$$

$$(4) f(x) = \frac{1}{\sqrt{x^2+5x-24}}$$

Problem 7. For each of the following pairs of functions, compute a) $f + g$, b) $f - g$, c) $f \cdot g$, d)

$\frac{f}{g}$:

$$(1) f(x) = 5x - 3, g(x) = 4x - 7$$

$$(2) f(x) = x - 5, g(x) = x^2 - 9x + 20$$

$$(3) f(x) = \frac{1}{4x+1}, g(x) = \frac{1}{3x-2}$$

Problem 8. For each of the following pairs of functions, compute $f \circ g$:

$$(1) f(x) = x^2 + 2x - 10, g(x) = 3x - 1$$

$$(2) f(x) = \frac{x-3}{x+2}, g(x) = \frac{2x-1}{x-2}$$

Problem 9. For each of the following functions, find $f^{-1}(x)$.

$$(1) f(x) = 7x - 10$$

$$(2) f(x) = \frac{8x-2}{3x+7}$$

Problem 10. Let f and g be functions that have compositional inverses f^{-1} and g^{-1} . Using the table below, answer the following questions:

x	$f(x)$	$g(x)$
1	2	3
2	1	4
3	4	1
4	3	2

TABLE 1

(1) $(f^{-1} \circ g \circ f)(3)$

(2) $(g \circ g^{-1} \circ f^{-1})(4)$

Problem 11. Let $f(x)$ and $g(x)$ both be odd functions. If $h(x)$ is a function with:

$$h(x) = f(x) \cdot g(x),$$

prove that $h(x)$ must be an even function. Hint: What is $h(-x)$?

Bonus Problems

Problem 12. Let $f(x)$ and $g(x)$ both be odd functions. If $h(x)$ is a function with:

$$h(x) = \frac{f(x) + g(x)}{f(x) \cdot g(x)},$$

is $h(x)$ an even function, an odd function, or neither? Show your work to receive credit; an answer with no work to support it will not get any points. Hint: This is similar to Problem 10.

Problem 13. Consider the function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } 0 \leq x < 5 \\ 3x + 1 & \text{if } x \geq 5 \end{cases}$$

- (1) If f were an even function, what is $f(-3)$?
- (2) If f were an odd function, what is $f(-7)$?